Capillary pressure and relative permeability estimation for low salinity waterflooding processes using pore network models

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Abstract

In this work, the effect of low salinity waterflooding (LSWF) on capillary pressure and relative permeability is studied at pore scale. For this purpose, a pore-scale model implemented on OpenPNM is developed to describe fluid flow, salinity transport and wettability change in a pore network. The effect is studied by considering high salinity waterflooding followed by low salinity waterflooding. The system wettability change is introduced as modifications of the local contact angle that are induced by the water salinity reduction. These modifications are evaluated by calculating the contact angle change in each pore and throat of the network, and then integrating it. From this analysis, a methodology has been developed to study the LSWF impact on capillary pressure and relative permeability. This methodology has been successfully applied to two pore network cases available in an open rock database; one network correspond to a sandstone and the other to a carbonate sample. The methodology developed here can be seen as a tool to complement laboratory tests needed to determine the efficiency of LSWF processes.

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Preprint submitted to Journal of Petroleum Science and Engineering April 8, 2019
Keywords: Low salinity waterflooding, pore network modeling, pore scale modeling, capillary pressure, relative permeability

1. Introduction

Low salinity waterflooding (LSWF) is an enhanced oil recovery (EOR) technique that mobilizes oil by reducing the salinity or modifying the composition of the injected water. The LSWF is considered as a simple and economical oil recovery technique due to the easy access to low salinity water sources, such as seawater (Al-Shalabi & Sepehrnoori, 2016; Nasralla et al., 2018). The LSWF potential was first recognized by Yildiz & Morrow (1996) and Tang & Morrow (1997), when they observed additional oil recovery in lab sandstone core floodings that has depended on injected water composition. Since then, LSWF has been successfully proven in laboratory flooding tests in sandstone and carbonate cores (Yildiz & Morrow, 1996; Tang & Morrow, 1997; Austad, 2013; Sheng, 2014; Sohal et al., 2016; Al-Shalabi & Sepehrnoori, 2016; Mahani et al., 2017), as well as in multiple sandstone reservoirs (Webb et al., 2004, 2005; Jerauld et al., 2008; Yousef et al., 2012; Aladasani et al., 2014). In many results, the oil recovery could be increased when injected water salinity was much lower than formation water salinity. Different mechanisms to explain the low salinity effect have been proposed (Austad, 2013; Sheng, 2014; Mahani et al., 2017). However, there is not yet a consensus on dominant LSWF mechanisms, and answers to many physics and chemistry questions are still pending (Sheng, 2014; Al-Shalabi & Sepehrnoori, 2016; Etemadi et al., 2017).

To date, several authors have simulated the LSWF process in lab cores (Jerauld et al., 2008; Wu & Bai, 2009; Mahani et al., 2011; Omekeh et al., 2012; Dang et al., 2013; Al-Shalabi & Sepehrnoori, 2016; Coronado & Díaz-Viera, 2017; Sanaei et al., 2018) using a mechanistic approach that parametrically modifies the relative permeability and capillary pressure curves as a function of salinity (Sheng, 2014). In all of these models, diverse macroscopic properties for the involved oil recovery mechanism were assumed. For example, they consider

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spatially homogeneous rock-fluid properties, such as surface tension and contact angle, that importantly affect the capillary pressure and relative permeability curve behavior, which are key inputs in simulations. An alternative approach to evaluate the change on the capillary pressure and relative permeability due to LSWF is modeling the fluid flow and salinity transport at pore scale, which is the purpose of this work.

In general, pore-scale modeling is constituted by three main components: (i) description of the geometry and structure of the porous media, (ii) definition and solution of the governing equations, and (iii) selection of the macroscopic properties (Raoof, 2011). In this way, it is possible to incorporate porous medium properties that are underestimated at larger scales. Further, pore-scale modeling discretizes the porous structure by heterogeneously distributing voids with fluid phases inside, and its approaches are potentially attractive tools for dynamic property estimation, such as pore networks. Pore network modeling comprises various disciplines, such as spatial statistics, imaging and mathematical modeling, which contribute to characterize the desired phenomena, as is well described in examples given by M. Blunt and coworkers (Blunt et al., 2002; Okabe & Blunt, 2004; Valvatne et al., 2005; Gharbi & Blunt, 2012; Mostaghimi et al., 2012; Blunt et al., 2013; Raeini et al., 2017; Xie et al., 2017; Raeini et al., 2018). Part of Blunt’s vast work focuses on imaging techniques and three-dimensional reconstructions applied in porous media. With these reconstructions and their coupling with mathematical models, Okabe & Blunt (2004); Valvatne et al. (2005); Mostaghimi et al. (2012) and Blunt et al. (2013) have studied Newtonian and non Newtonian flow in several rock samples (Okabe & Blunt, 2004; Valvatne et al., 2005; Mostaghimi et al., 2012; Blunt et al., 2013). In the same way, the pore network modeling application to other research areas includes: CO₂ sequestration (Kim & Santamarina, 2015), methane hydrate gas production (Mahabadi & Jang, 2014; Jang & Santamarina, 2014; Mahabadi et al., 2016a), gas bubble nucleation/migration (Zhao & Ioannidis, 2011), water retention curves (Dai & Santamarina, 2013; Kang et al., 2016; Mahabadi et al., 2016b), drying process in porous media (Yiotis et al., 2001), and polymer ele-
trolyte membranes for fuel cells (Gostick et al., 2006, 2007; Gostick & Weber, 2015).

In the last two decades, EOR modeling via pore networks has grown importantly and acquired great interest. In principle, this is due to two main factors: the computational developments and the improvement in the detailed description rock-fluid systems. With a proper physical-mathematical formulation, the pore scale is able to capture some of the proposed EOR mechanisms that are not yet fully understood at larger scales, such as wettability alteration in rock-fluid systems. Bolandtaba & Skauge (2011) developed a network to study residual oil mobilization by polymer injection, where adsorption particle, entrapment by the polymer, and viscous forces were considered (Bolandtaba & Skauge, 2011).

Hammond & Unsal (2012) and Qin & Hassanizadeh (2015) studied different mechanisms for microbial enhanced oil recovery, where the surfactants in the rock-fluid system produced oil mobilization and wettability change (Hammond & Unsal, 2012; Qin & Hassanizadeh, 2015). Lu & Yortsos (2001) established a model to simulate in situ combustion of forward filtration, where the porous microstructure and the solid fuel distribution effects on filtering combustion dynamics were investigated (Lu & Yortsos, 2001). Xu et al. (2018) modeled multiple physicochemical and thermal processes in combustion fronts in order to reduce potential risks in practical applications, such as high combustion temperature and low oxygen rate (Xu et al., 2018). Concerning the LSWF problem, Sorbie & Collins (2010) proposed a detailed semiquantitative theory, based on theoretical considerations at pore scale, of how the LSWF effect works (Sorbie & Collins, 2010). Subsequently, Watson et al. (2011) addressed an uncertainty assessment on wettability alteration, which is one of the most discussed LSWF mechanisms. The generated network allows one to carry out a systematic investigation of oil-water-rock parameters which are critical for oil recovery (Watson et al., 2011). Boujelben et al. (2018) described a dynamic model to investigate LSWF effects on oil recovery under dynamic flow conditions. Here, the salinity spatial distribution is tracked explicitly during the recovery process, and the fluid distribution is updated according to a relative balance between capillary
and viscous forces. Further, the capillary effects are correlated to salinity by relating contact angle and local injected water concentration (Boujelben et al., 2018).

In this paper a systematic methodology to simulate the LSWF process at pore scale is presented. This work is one of the first developments made to model LSWF using pore networks. The methodology will provide a tool to observe the salinity effects on effective flow properties, and using it the LSWF effect on capillary pressure and relative permeability curves is studied. This paper is structured as follows. First, the pore network modeling details are presented. Second, the proposed methodology for LSWF is discussed. Finally, two cases that show the methodology are presented. In these cases, fluid flow, salinity transport, and primary drainage are recreated to estimate absolute permeability, capillary pressure, and relative permeability curves.

2. Pore network modeling

Pore network modeling is an approach to study a wide range of phenomena, taking into account both the medium's geometry and its connectivity. One of the main applications is to estimate effective flow and transport properties, such as capillary pressure curves, effective permeabilities, and effective diffusion coefficient, among others, which regularly are obtained by laboratory tests and experimental correlations.

The pore network modeling discretizes the porous medium morphology which is composed by its internal structure and its topology (the way in which the medium is connected) (Sahimi, 2011). In principle, the internal structure, which represents porosity, is constituted by pores and throats. The pores are defined as the larger voids connected by narrower paths called throats (Blunt et al., 2013). In general, the key parts of a pore network model are the pore structure and the equations describing the phenomena, according to the study scale. The first part is commonly known as geometry, which expresses the medium connectivity and discretizes throats and pores by means of geometric entities
(Aker et al., 1998). The second part is the mathematical modeling that describes the phenomena in porous media. Coupling these elements adequately will allow optimal estimates (Arns et al., 2004; Martínez-Mendoza, 2016). In this paper, these two main elements are decomposed in order to gain a better overall model understanding. The elements involved are: (i) network, (ii) geometry, (iii) phases, (iv) pore-scale physics, and (v) methods (see Figure 1). The development of these five elements will enhance the overall pore network model. Briefly, each pore network element is described below.

Figure 1: In general, the key elements of a pore network model are the medium structure and the mathematical models, and their adequate coupling can describe the phenomena at pore scale. In this work, these main elements are decomposed into: (i) network, (ii) geometry, (iii) phases, (iv) pore-scale physics, and (v) methods.

2.1. Network

It is the set of spatially localized void sites connected by bonds (Raoof et al., 2013). The network is the element that denotes the medium connectivity, it supports the model geometry and serves as simulation mesh. An important network property is the coordination number. The coordination number $z$ de-
scribes the connectivity between sites and bonds, which means the number of bonds connected to a site.

2.2. Geometry

Geometric entities placed in the network shape the internal medium structure. The network sites support the pores, while the bonds the throats. These elements can have both regular and irregular forms, being the former the one that offers a larger capability to describe the physical process. The irregular shapes can extend the study scope over processes that in a larger scale are negligible (Sahimi, 2011).

2.3. Phases

Depending on the process to be recreated, fluid or solid phases must be physically and chemically described by constitutive equations or correlations.

2.4. Pore-scale physics and methods

To set the phase behavior at pore scale, the pore and throat geometry has to work together with proper definitions, physical laws, and constitutive equations; thus, suitable expressions for pore network models are obtained. All this is known as pore-scale physics (Ioannidis & Chatzis, 1993; Gostick et al., 2007). By employing these assumptions, it becomes easier to establish methods that describe the nature of the desired events. Here, the phenomena of interest are fluid flow, salinity transport, and immiscible displacement.

3. Fluid flow and salinity transport modeling in pore networks

The LSWF process will be modeled as a decoupled flow and transport problem. From the fluid flow problem, pore pressure values will be computed, and they will be used in the transport problem to estimate the salinity concentration in both pores and throats. The concentration values will shape the salinity state in the network.
3.1. Conceptual model

The mathematical model considers a porous medium discretized by pores that are connected by narrower paths called throats, whose pore and throat geometry is invariant in time. Further, it is assumed that, during the whole process the porous medium is completely occupied by a fluid phase which contains only one dissolved component. The fluid flow and salinity transport will be solved in a decoupled way. The fluid flow through the network is under saturated conditions assuming laminar, Newtonian and single-phase flow. After imposing a pressure gradient between two opposite boundaries, a flow field in the network will take place. The salinity transport is inside the system is considered to follow advective-diffusive processes. Each network element (pores and throats) will be taken as a control volume, and salinity mass balance is solved over each element. Finally, a fully mixed domain is considered both in pores and in throats, therefore, a constant salinity gradient is assumed.

3.2. Mathematical formulation

3.2.1. Fluid flow

The fluid flow through pore network models assumes the Hagen-Poiseuille law, which describes laminar flow, $q$, of an incompressible and Newtonian fluid, under a pressure drop effect:

$$q = -\frac{\pi r^4 \Delta p}{8 \mu l}$$

where $r$ is the conduit radius, $l$ is the conduit length, $\mu$ is the fluid viscosity, and $p$ is the pressure. From this expression, the fluid hydraulic conductance $\kappa$ follows as:

$$\kappa = \frac{\pi r^4}{8 \mu l}$$

Considering a mass balance over each pore in the network, and using Equation 1, yields:

$$\sum_{j=1}^{n} \kappa_{ij} (p_i - p_j) = 0$$

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where $\kappa_{ij}$ is the total hydraulic conductance between pore $i$ and pore $j$, whilst $p_i$ and $p_j$ are the pressures at each element. Equation 3 denotes the fluid flow model for pore networks. The total hydraulic conductance is computed as the hydraulic conductance sum through half of pore $i$, the connecting throat $ij$, and half of pore $j$. This parameter ($\kappa_{ij}$) depends on the size and the length of pores and throats.

Equation 3 sets up the system of algebraic equations for the pressure value at each pore of the network to be solved. By this way the equation set $Ax = b$ is obtained, where $A$ is a banded matrix constituted by the hydraulic conductivities $\kappa_{ij}$; $x$ is the pore pressure vector, and $b$ contains the boundary condition values. After solving the system, the pore network model permeability is obtained by Darcy’s law:

$$K = \frac{\mu QL}{A(p_{in} - p_{out})}$$

(4)

where $Q$ is the total flow across the network, $A$ is the cross-sectional area normal to flow direction, $L$ is the network length in the flow direction, $p_{in}$ is the inlet pressure, and $p_{out}$ is the outlet pressure.

### 3.2.2. Salinity transport

From the fluid flow problem, the pore pressure values are used to model the salinity advance via advective-diffusive transport.

For a given pore $i$, the one-component transport equation is:

$$V_i \frac{dc_i}{dt} + \sum_{j \in I_i} z_i q_{ij} c_i - \sum_{j \in I_i} z_i q_{ij} c_{ij} = \sum_{j \in I_i} D_e A_{ij} \frac{c_{ij} - c_i}{l_{ij}}, \quad \forall \ i \in [1, N_p]$$

(5)

where $V_i$ and $c_i$ are the volume and the salinity concentration for pore $i$, respectively. The flow rate and the salinity concentration throughout throat $ij$ are denoted as $q_{ij}$ and $c_{ij}$, respectively. $D_e$ is the effective diffusion coefficient, $A_{ij}$ is the connecting throat cross-sectional area, and $l_{ij}$ is the connecting throat length. The total number of pores in the network is expressed as $N_p$, $z_i$ is the pore coordination number, and $I_i$ is the index set containing all the indexes of the pores connected to pore $i$ through throat $ij$. 

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In the same way, the salinity transport equation for a given throat $ij$ is:
\[
V_{ij} \frac{dc_{ij}}{dt} + q_{ij}c_i + q_{ij}c_j - 2q_{ij}c_{ij} = D_eA_{ij}\frac{c_i - c_{ij}}{l_{ij}} + D_eA_{ij}\frac{c_j - c_{ij}}{l_{ij}}, \quad \forall \ i \in [1, N]
\]
where $V_{ij}$ is the volume and $c_{ij}$ is the salinity concentration applying to throat $ij$, respectively, and $c_j$ is the salinity concentration within pore $j$.

Equation 5 and Equation 6 comprise the mathematical model of fluid flow and salinity transport for pore networks, and their corresponding initial and boundary conditions are given by:
\[
c(I, t_0) = c_0
\]
\[
c(I_{inlet}, t \geq t_0) = c_{inlet}
\]
\[
\frac{dc}{dx} \bigg|_{I_{outlet}, t \geq t_0} = 0
\]
where $I$ is the set of pore and throat indexes. $I_{inlet}$ and $I_{outlet}$ are the indexes of pores placed at the inlet and the outlet boundaries, respectively. The initial salinity concentration is denoted as $c_0$, while the salinity concentration at the inlet is $c_{inlet}$. The initial time is expressed as $t_0$.

### 3.3. Numerical discretization

A first order backward finite difference is applied in the numerical solution, therefore, a fully implicit solution scheme is obtained. For a given pore $i$, the discretized Equation 5 can be written as:
\[
V_i\frac{c_i^t - c_i^{t-1}}{\Delta t} = \sum_{j \in I_i} q_{ij}c_{ij}^t - \sum_{j \in I_i} q_{ij}c_{ij}^t + \sum_{j \in I_i} D_eA_{ij}\frac{c_{ij}^t - c_{ij}^{t-1}}{l_{ij}}
\]
where the time step is denoted as $\Delta t$. Leaving only $c_i^{t-1}$ on the left hand side of the Equation 8, results:
\[
c_i^{t-1} = c_i^t - \Delta t \frac{V_i}{\sum_{j \in I_i} q_{ij}c_{ij}^t - \sum_{j \in I_i} q_{ij}c_{ij}^t + \sum_{j \in I_i} D_eA_{ij}\frac{c_{ij}^t - c_{ij}^{t-1}}{l_{ij}}}
\]
Similarly, the numerical discretization of Equation 6 for a given throat $ij$ is:
\[
c_{ij}^{t-1} = c_{ij}^t - \frac{\Delta t}{V_{ij}} \left[ q_{ij}c_{ij}^t + q_{ij}c_{ij}^t - 2q_{ij}c_{ij}^t + D_eA_{ij}\frac{c_{ij}^t - c_{ij}^{t-1}}{l_{ij}} + D_eA_{ij}\frac{c_{ij}^t - c_{ij}^{t-1}}{l_{ij}} \right]
\]
The discretized initial and boundary conditions applied to Equation 9 and Equation 10 are:

\[ c_i = c_0, \quad \forall i \in I_{pore}, \quad t = t_0 \]  
\[ c_{ij} = c_0, \quad \forall ij \in I_{throat}, \quad t = t_0 \]  
\[ c_i = c_{inlet}, \quad \forall i \in I_{inlet}, \quad t \geq t_0 \]  
\[ \sum_{j \in I_{outlet}} \frac{c_{ij}^t - c_i^t}{t_{ij}} = 0, \quad t \geq t_0 \]  

where \( I_{pore} \) and \( I_{throat} \) are the indexes of pores and throats, respectively. Equation 9 and Equation 10 lead to a linear system \( Ax = b \), where the number of unknown variables is equal to the total number of pores and throats \( N_p + N_t \).

Additionally, for scheme stability, the minimum time step is chosen on the basis of throat residence times (Raoof, 2011; Acharya et al., 2005):

\[ \Delta t = \min \left\{ \frac{|V_{ij}|}{q_{ij}^{-1}} \right\} = \min \{ T_{ij} \} \]  

In Equation 12, \( T_{ij} \) expresses the fluid residence time within throat \( ij \). Since the advective-diffusive transport processes drive the salinity fluxes, the smallest throat, with higher flux rate, sets the time step (Raoof et al., 2014).

### 3.4 Computational implementation

The salinity transport model was implemented in OpenPNM which is an open-source pore network modeling package coded in Python. This package is fully independent of the network topology and dimensionality, since graph theory descriptors to represent the structure is used (Gostick et al., 2016). Furthermore, OpenPNM is designed to be customized; that is, users can code their own pore-scale physics and thermophysical property models (Fazeli et al., 2016; Sadeghi et al., 2017; Tranter et al., 2016, 2018). The validation of our implementation has been made against an 1D convective-dispersive solute transport equation provided by van Genuchten & Alves (1982) as described in Appendix A.
4. Workflow to study a LSWF process

In this work the LSWF modeling involves evaluating the salinity impact on capillary pressure and relative permeability. By following a pore network based methodology, which is described in Appendix B, a pore-scale physics to recreate a LSWF process is established (Martínez-Mendoza, 2018). The workflow to study LSWF processes is shown in Figure 2 and listed as follows:

![Workflow diagram](image)

Figure 2: Workflow using the pore network approach to investigate low salinity waterflooding.

1. Pore network model: the network is mainly generated from imaging techniques. The open access data from two generic samples is used to set two case studies. The selection was based on the absolute permeability and the porosity values of the generic samples are close to information from experimental tests of our interest. The experiments study the injection of water with different salinity composition and its impact on oil recovery. Since these experiments were performed at lab scale, we only use their petrophysical information and description of the fluids to set our pore-scale models.
2. Fluid flow: to describe flow in the network, the model in Equation 3 is used, and the simulation parameters and conditions were taken from the experimental tests. After solving this equation, the pressure in each pore and the flow rate in throats are obtained.

3. Salinity transport: by using the previous pressure values, the salinity transport equations (Equation 5 and Equation 6) are solved. As a result, the salinity concentration, at different simulation times, at each pore and throat in the network is obtained. This information will allow the evaluation of the injection of water with different salinity composition. It should be noted that, since salinity transport effect on fluid flow is negligible, the pressure field should be computed only once, at the workflow start.

4. Contact angle evaluation: the wettability is evaluated by considering a linear relationship between contact angle ($\theta$) and salinity concentration ($c$). This evaluation is an attempt to assess the salinity impact on contact angle.

5. Capillary pressure: a primary drainage process is simulated, at each time step of the salinity transport problem, in which an interest oil is the invading fluid and the injected water is the defending phase. After determining the invasion sequence, i.e. the water-invaded pores and throats, the capillary pressure curves for oil and water systems are estimated.

6. Relative permeability: the relative permeability curves are computed from the primary drainage invasion sequence and considering a capillary bundle model.

4.1. Salinity effect on contact angle

Wettability is the preferred tendency of a fluid to spread or adhere to a solid surface, in the presence of another immiscible fluid. A parameter that reflects such a preference is the contact angle $\theta$. In pore-scale modeling, several mechanisms for wettability behavior have been proposed. In this work, the wettability change, and therefore the contact angle, is assumed as the principal
consequence of LSWF effects. In principle, the throat initial contact angle value could be changed when low salinity water invades the throat. As a result, reduction in capillary forces, that could affect oil recovery, would be expected if the contact angle change modifies the wettability to a less oil-wet system (Boujelben et al., 2018). The following linear relationship between contact angle and salinity concentration is assumed (Aladasani et al., 2014):

\[
\theta(c) = \theta^{HS} - \frac{c}{c^{HS} - c^{LS}} \left( \theta^{HS} - \theta^{LS} \right)
\]

where, \(\theta^{HS}\) is the contact angle at the high salinity concentration, \(c^{HS}\), and \(\theta^{LS}\) is the contact angle at the low salinity concentration, \(c^{LS}\).

### 4.2 Relative permeability

From the primary drainage capillary pressure and saturation data, we estimate the relative permeability curves using a bundle model. We consider the Rodríguez & Teyssier (1974) model which reproduces the displacement of a wetting fluid by a non wetting fluid in porous media (Rodríguez & Teyssier, 1974). The model can be written as:

\[
j = \frac{\log \left( \int_{s_{wi}}^{s_w} \frac{ds_w}{p_c} / \int_{s_{wi}}^{1} \frac{ds_w}{p_c} \right)}{\log \left( \frac{s_w - s_{wi}}{1 - s_{wi}} \right)}
\]

\[
k_{rw} = \left( \frac{s_w - s_{wi}}{1 - s_{wi}} \right)^{2+j}
\]

\[
k_{rhw} = \left[ 1 - \frac{s_w - s_{wi}}{1 - s_{wi}} \right]^{2} \left[ 1 - \left( \frac{s_w - s_{wi}}{1 - s_{wi} - s_{nwi}} \right)^{j} \right]
\]

where \(p_c\) is the capillary pressure, \(s_w\) is the wetting phase saturation, and \(s_{wi}\) is the irreducible wetting phase saturation. The subscript \(nw\) stands for the non wetting phase. Finally, \(k_{rw}\) and \(k_{rhw}\) refer to the wetting and the non wetting phase relative permeability, respectively.

### 5. Application to case studies

In this section the methodology is applied to two LSWF cases, one for sandstones and the other for carbonates. In the first case the capillary and relative
permeability curves are calculated and fitted for a sandstone-oil-water system. LSWF test information and fluid properties were taken from an experimental test, while the rock sample characteristics were taken from literature. In the carbonate second case, data from a LSWF lab test and open access information were used. The objective of these applications are: 1) demonstrate the model capacity to simulate a LSWF process at pore scale, and 2) observe the salinity effect on capillary pressure and relative permeability curves. The applications are described below.

5.1. Case 1: LSWF in a sandstone sample

This case seeks to introduce our methodology and shows a pore-scale LSWF study in sandstones. We used a laboratory-scale test in a sandstone core, where additional oil recovery is observed after a sequential injection of formation water and 100-times diluted formation water (Yousef et al., 2011). Further, open access information of a sandstone sample from a Middle East reservoir is used to generate the pore network model (Dong et al., 2007). However, not having the corresponding effective property measurements during the LSWF test (capillary pressure and relative permeability), our results establish an exploratory analysis on the expected values. The experimental test properties for Case 1 are summarized in Table 1.

In addition to the experimental test, we use the open access information of a generic sandstone sample from an Arabian reservoir (ICL, 2014b), which is here called Sample S1. Dong (2007) performed micro-computed tomography scanning of sandstone Sample S1 and several other rock samples (Dong, 2007). In this section, we use the extracted network and the pore and throat sizes from sandstone Sample S1, since its porosity and permeability values are close to those found in the sample of the experimental test. Details about the network model are presented in Table 2.

A network of 1717 pores and 2824 throats discretizes the sandstone Sample S1, and it is used in the next computations. Pores are assumed to be spheres, while throats are represented as cylinders. Both entities share a circular cross-
Table 1: Information from an experimental test and Yousef et al. (2011) to set the case studies.

<table>
<thead>
<tr>
<th>Property</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core length, $L$</td>
<td>0.049</td>
<td>0.041</td>
<td>m</td>
</tr>
<tr>
<td>Core diameter, $d$</td>
<td>0.038</td>
<td>0.038</td>
<td>m</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.18</td>
<td>0.25</td>
<td>$m^3/m^3$</td>
</tr>
<tr>
<td>Absolute permeability, $k$</td>
<td>6.46E-14</td>
<td>3.91E-14</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Temperature, $T$</td>
<td>363.15</td>
<td>373.15</td>
<td>K</td>
</tr>
<tr>
<td>Pressure, $p$</td>
<td>17.2</td>
<td>12.4</td>
<td>MPa</td>
</tr>
<tr>
<td>Cross-sectional area, $A$</td>
<td>1.14E-03</td>
<td>1.14E-03</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Volume, $V$</td>
<td>5.71E-05</td>
<td>4.63E-05</td>
<td>$m^3$</td>
</tr>
<tr>
<td>Pore volume, $PV$</td>
<td>1.05E-05</td>
<td>1.16E-05</td>
<td>$m^3$</td>
</tr>
</tbody>
</table>

The LSWF simulation considers two fluids that are injected sequentially into the network, which are formation water and formation water diluted 100 times. The formation water is the first fluid invading the network and has a high salinity.
Table 2: Description of the pore network models used in case studies.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone Sample</td>
<td>Irregular</td>
<td>Irregular</td>
</tr>
<tr>
<td>Sample S1</td>
<td>1717 pores</td>
<td>4311 pores</td>
</tr>
<tr>
<td></td>
<td>2824 throats</td>
<td>7688 throats</td>
</tr>
<tr>
<td></td>
<td>Volume = 1.76E-8 $m^3$</td>
<td>Volume = 9.77E-9 $m^3$</td>
</tr>
<tr>
<td></td>
<td>Porosity = 14.35 (%)</td>
<td>Porosity = 24.41 (%)</td>
</tr>
<tr>
<td></td>
<td>Absolute permeability = 1.53E-12 $m^2$</td>
<td>Absolute permeability = 3.76E-14 $m^2$</td>
</tr>
</tbody>
</table>

- **Geometry**: Pores: spheres, Throats: cylinders, Cross section: circular
- **Pore/throat sizes**: from imaging and from distributions

- **Physics**: Capillary pressure: Young-Laplace, Hydraulic conductivity for cylinders

concentration (HS). The low salinity fluid (LS) is the formation water diluted 100 times. The contact angle limit values for high and low salinity were taken from Moustafa et al. (2015). Moustafa et al. (2015) investigated the effect of several nanomaterials on contact angle for a sandstone sample (Moustafa et al., 2015). Table 3 summarizes the fluid properties used in Case 1.

Table 3: Fluid properties used in Case 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Oil</th>
<th>Formation water diluted 100 times</th>
<th>Formation water</th>
<th>Unit</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>881.6</td>
<td>1,130.0</td>
<td>1,013.8</td>
<td>$kg/m^3$</td>
<td>Experiment</td>
</tr>
<tr>
<td>Viscosity</td>
<td>1.08E-02</td>
<td>3.0E-04</td>
<td>4.85E-04</td>
<td>$Pa.s$</td>
<td>Experiment</td>
</tr>
<tr>
<td>Interfacial tension</td>
<td>—</td>
<td>0.0234</td>
<td>0.0172</td>
<td>$N/m$</td>
<td>Experiment</td>
</tr>
<tr>
<td>Contact angle</td>
<td>—</td>
<td>72</td>
<td>65</td>
<td>$(°)$</td>
<td>Moustafa et al. (2015)</td>
</tr>
<tr>
<td>Salinity concentration</td>
<td>—</td>
<td>216,000</td>
<td>2,160</td>
<td>$ppm$</td>
<td>Experiment</td>
</tr>
</tbody>
</table>

In the simulation, we solved the fluid flow and salinity transport problem in an uncoupled way; first the fluid flow is simulated in the network, and with the
Table 4: Parameters considered for LSWF process simulation in Case 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HS</th>
<th>LS</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection time ($t_{inj}$)</td>
<td>7,000</td>
<td>7,000</td>
<td>s</td>
</tr>
<tr>
<td>Time step ($\Delta t$)</td>
<td>1</td>
<td>1</td>
<td>s</td>
</tr>
<tr>
<td>Initial salinity concentration ($c_0$)</td>
<td>$c^H_{final}$</td>
<td>ppm</td>
<td></td>
</tr>
<tr>
<td>Salinity concentration at inlet ($c_{in}$)</td>
<td>216,000</td>
<td>2,160</td>
<td>ppm</td>
</tr>
<tr>
<td>Neumann condition at outlet</td>
<td>0</td>
<td>0</td>
<td>ppm/m</td>
</tr>
</tbody>
</table>

resulting pressure field the salinity transport is then computed. The LSWF process begins invading the network with formation water, during a given injection time $t_{inj}$. Later, the diluted water is injected to start the recovery stage. Therefore, at the simulation ending, different concentration states for each time step are obtained; salinity concentration values are known in each pore and throat. Our simulation considers a four-hour LSWF process, 7000 [s] each stage; additional information regarding the simulation is shown in Table 4. In general, during the first stage the salinity concentration in the network increases until a steady-state at high salinity is reached. By starting the low salinity stage, the network concentration decreases, and most of the network elements reach the LS condition. Figure 4 displays the salinity in the network during the HS and LS stages, at two simulation times. The salinity concentration values are used to calculate the throat contact angle, where Equation 13 is used, it shows that as salinity decreases, contact angle also does. The contact angle change modifies the network wettability to a less oil-wet system. At the LSWF process end, the network has a homogeneous contact angle distribution. In addition, there are no abrupt $\theta$ changes in throats. In the HS stage a value equal to 72° is reached, while in LS stage the value is 65°. Considering the contact angle as an amount that denotes the wettability degree, we find that the sandstone’s wettability, in HS and LS, is homogeneous. The latter can favor the residual oil displacement and, thereby, increase the oil recovery factor.
Figure 4: Salinity concentration in sandstone Sample S1 network during LSWF simulation for Case 1. HS stage simulation at (a) 700 and (c) 7000 seconds. LS stage simulation at (b) 700 and (d) 7000 seconds. The water flows from left to right.

According to the limiting contact angle values for the high and low salinity states (72° and 65°, respectively), the system is slightly intermediate wet when the HS fluid is the invading phase. After we switch to the LS fluid, a wettability change in the network is observed. At the LSWF process end, the system has gained stronger affinity to water. The corresponding capillary pressure curves are shown in Figure 5. The red curve in Figure 5(a) gives the LS initial condition, where the capillary pressure values range from 0.2 to 12 [kPa]. When the LS stage finishes (curve in blue), the irreducible water saturation is almost the same as the initial condition, with a value equal to 7%. However, higher pressure values are required to displace oil when the LSWF ends. It follows that the
LSWF gives place to a slightly wettability modification in the network, because the LSWF produces an increase in preference towards water. Subsequently, we fitted a Brooks-Corey model to the initial and final LS curves. The fitted models can be used in simulations at larger scales. Table 5 summarizes the fitted Brooks-Corey parameters, and plots Figure 5(b) and Figure 5(c) show the fitted curves and the original data points.

![Capillary pressure curves for Case 1. (a) Capillary pressure curves showing the LSWF effect. The red curve is for the HS, while the blue curve for LS. In (b) and (c) the fitted Brooks-Corey \( p_c(s_w) \) models for HS and LS, respectively. The blue points represent the values calculated in the primary drainage algorithm. The orange points are obtained using the fitted model. The dashed orange line shows the fitted curve trend.](chart.png)
Table 5: Fitting parameters for capillary pressure and relative permeability in HS and LS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary pressure exponent</td>
<td>4.05</td>
<td>3.62</td>
</tr>
<tr>
<td>Entry pressure [Pa]</td>
<td>536</td>
<td>683</td>
</tr>
<tr>
<td>Water endpoint ($k_{rw}^0$)</td>
<td>0.35</td>
<td>0.65</td>
</tr>
<tr>
<td>Oil endpoint ($k_{ro}^0$)</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>Water exponent ($n_w$)</td>
<td>3.36</td>
<td>3.77</td>
</tr>
<tr>
<td>Oil exponent ($n_o$)</td>
<td>1.69</td>
<td>1.91</td>
</tr>
<tr>
<td>Residual water saturation ($s_{wr}$)</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Residual oil saturation ($s_{or}$)</td>
<td>0.25</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Figure 6(a) shows the estimated relative permeability curve, the solid lines are for HS and the dashed for LS. In the HS period, a residual oil saturation equal to 0.25 is obtained. The $k_r$ curve crossing takes place when the water saturation $s_w^{cross}$ is 0.53, in which $k_{rw}^{cross} = k_{rw}^{cross} = 0.11$. Further, the water and oil relative permeability endpoint is 0.32 and 0.97, respectively. For $s_w < 0.4$, $k_{rw}$ values are almost zero, this points to a mixed wet system. In the LS stage, these properties undergo minimal changes. The $s_{or}$ value decreases 60 percent, $s_{or} = 0.1$. The $k_r$ curve crossing point occurs at a $s_w^{cross} = 0.57$, with $k_r = 0.11$. The curve endpoints are 0.96 and 0.65 for oil and water, respectively. This behavior is consistent with the wettability change in capillary pressure. Finally, from the stage end, we fitted a Brooks-Corey $k_{rw}$ model and a modified Brooks-Corey $k_{ro}$ model to the estimated curves. The fitted curves are shown in Figure 6(b) and Figure 6(c), and their respective parameters are summarized in Table 5.
Figure 6: Relative permeability curves for Case 1. (a) The red lines show the oil relative permeability $k_{ro}$ and the blue lines the water relative permeability $k_{rw}$. Solid lines correspond to HS and dashed lines to LS, and original data are in red and blue. The curves were fitted at the end of the (b) HS and (c) LS stages. In curve fitting, a Brooks-Corey $k_{ro}$ model (green line) and a modified Brooks-Corey $k_{ro}$ model (pink line) were used.

5.2. Case 2: LSWF in a carbonate sample

Through this case, we evaluate the LSWF impact on capillary pressure and relative permeability curves for a carbonate rock sample. Here we employ laboratory data from Yousef et al. (2011) and take the data for connate water and seawater for HS and LS, respectively, as well the data for viscosity, interfacial tension, contact angle, and salinity concentration. We use open access
information from a generic carbonate sample described as C2 (see Figure 7(a)) (ICL, 2014a), and we fitted probability distribution functions for C2 pore and throat sizes. Subsequently, the fitted distribution parameters were modified, in order to reproduce the porosity and permeability values of Yousef et al. (2011). We straightforwardly use the C2 extracted network without any modification, keeping the network topology invariant, thus the proper carbonate topology is ensured. By using the extracted network in combination with the fitted probability distribution, a new carbonate sample version is obtained, which is labeled as C2-Yousef in Figure 7(b).

Figure 7: The synthetic carbonate Sample C2-Yousef used to simulate LSWF process in Case 2. (a) Micro CT image showing the carbonate C2 pore structure. (b) Pore network model C2-Yousef based on C2 structure. Spherical pores (red) and cylindrical throats (blue) are employed.

Figure 8 shows the probability distribution functions for pore and throat size comparison between the carbonate C2 and the synthetic sample C2-Yousef. Figure 8(a) presents the pore diameter histograms, and Figure 8(b) shows the corresponding histograms for throat diameters. The carbonate C2 pore/throat diameters are those obtained and reported by Dong (2007). On the other hand, the diameter values for the C2-Yousef network were generated by probability distributions. The pore and throat size distribution of sample C2-Yousef was established by fitting a distribution to the original carbonate C2 data and then disturbing the fitting parameters. The distribution parameter selection was
made in a deterministic way until reaching absolute permeability and porosity values similar to Yousef et al. (2011) It is important to note that no spatial dependence is considered in this process. Table 6 summarizes the probability distribution parameters for samples C2 and C2-Yousef.

Table 6: Probability distributions for pore and throat diameters in Case 2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Distribution</th>
<th>Shape</th>
<th>Location</th>
<th>Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2 pores</td>
<td>Lognormal</td>
<td>0.6</td>
<td>-1.5</td>
<td>11</td>
</tr>
<tr>
<td>C2-Yousef pores</td>
<td>Lognormal</td>
<td>0.2</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td>C2 throats</td>
<td>Lognormal</td>
<td>0.7</td>
<td>-0.3</td>
<td>5</td>
</tr>
<tr>
<td>C2-Yousef throats</td>
<td>Lognormal</td>
<td>0.9</td>
<td>1</td>
<td>30</td>
</tr>
</tbody>
</table>

The C2-Yousef network model consists of 8508 pores and 9818 throats, which are represented by spheres and cylinders, respectively. The model description is summarized in Table 2. In general, the network has highly dense areas of pores and throats, as well as considerable empty regions. This is attributed to the underlying complexity of carbonate pore structures. For this issue, image processing techniques have to treat microporosity and fractures in carbonates. In addition, if we wish to represent the porous structure by means of more complex pores and throats, it is necessary to take into account other geometric proper-

Figure 8: Pore and throat diameter histograms. (a) Comparison between C2 (green) and C2-Yousef (orange) pore diameters, and (b) the corresponding histograms for throat diameters.
ties, such as angularity, inscribed and effective diameter, and shape factor. This new information can only be obtained from high-resolution imaging techniques.

The simulation considers formation water with HS concentration (213,000 [ppm]), while seawater is the LS fluid, whose concentration is almost a quarter of that of HS (57,600 [ppm]). In addition, we use the same injection time and time step values from Case 1 (Table 4). The injected water information, for HS and LS, was taken from Yousef et al. (2011), whose fluid property values are displayed in Table 7.

<table>
<thead>
<tr>
<th>Property</th>
<th>Oil</th>
<th>Formation water</th>
<th>Seawater</th>
<th>Units</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>873.0</td>
<td>1,108.3</td>
<td>1,015.2</td>
<td>kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Viscosity</td>
<td>1E-03</td>
<td>4.76E-4</td>
<td>2.72E-04</td>
<td>Pa.s</td>
<td></td>
</tr>
<tr>
<td>Interfacial tension</td>
<td>—</td>
<td>0.0397</td>
<td>0.0339</td>
<td>N/m</td>
<td>Yousef et al. (2011)</td>
</tr>
<tr>
<td>Contact angle</td>
<td>—</td>
<td>92</td>
<td>80</td>
<td>°</td>
<td></td>
</tr>
<tr>
<td>Salinity concentration</td>
<td>—</td>
<td>213,000</td>
<td>57,600</td>
<td>ppm</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9 shows the salinity concentration in the network C2-Yousef during LSWF simulation. It is observed that, the water injection through the network is from left to right. According to the chosen time step, when the high salinity waterflooding (HSWF) ends, inlet pores and some neighboring pores reach the HS concentration (213,000 [ppm]), and a progressive concentration change in the network is observed. On the other hand, at the end of the LS process, most of the network has a concentration close to 60,000 [ppm]. It is noteworthy the existence of pores and throats with concentration values being almost zero in both recovery processes (HS and LS). Intuitively, this is attributed to small enough throat sizes to constrain advective transport effects. In principle, for this particular case, these pores and throats constitute no-flow zones which do not contribute for computing the absolute permeability.
(a) HS @ 4200 seconds.

(b) HS @ 7000 seconds.

(c) LS @ 4200 seconds.

(d) LS @ 7000 seconds.

Figure 9: Salinity concentration in C2-Yousef network during LSWF simulation for Case 2. HS stage simulation at (a) 4200 and (c) 7000 seconds. LS stage simulation at (b) 4200 and (d) 7000 seconds. The water flows from left to right.

By running primary drainage algorithm uncoupled from the salinity transport, capillary pressure curves for each time step are estimated, and they are shown in Figure 10. In this way, it is observe how the network salinity, which represents the LSWF process, displaces and modifies the capillary pressure curves. We link this event to a wettability change. At this scale, the wettability change impacts on modifying or reestablishing preferential flow zones, as well as an oil redistribution, since the capillary pressure in a given throat can be increased or decreased (Boujelben et al., 2018).

At the HS stage end, the C2-Yousef system remains with an intermediate wettability, but slightly preferring an oil contact; the maximum throat contact
angle value is $92^\circ > 90^\circ$. Figure 10(a) shows this effect, where the red curve is for the HSWF end, and in turn is the LSWF start. In this curve, capillary pressure values range from -5 to 25 [kPa]. This negative to positive value transition points out that oil prefers to be attached to the rock surface, when the system is totally saturated by water ($s_w \to 1$). As water saturation decreases, pressure increases and becomes positive. Therefore, the capillary pressure is positive and the wettability system changes from slightly oil wet to less oil wet, when $s_w < 0.8$.

Subsequently, when the LS stage is completed, the capillary pressure curve is positive and shows an even less oil wettability (curve in blue). In general, at the LSWF end, the system has a slight preference towards water; that is, a minor wettability alteration occurs due to the LSWF process. However, it is difficult to establish a clear and significant wettability change for this case study, given that the contact angle value range ($92^\circ - 76^\circ$) is for intermediate wettability. Figure 10(b) and Figure 10(c) show the fitted $p_c(s_w)$ models for HS and LS, respectively, and Table 5 summarizes the fitting parameters.
Figure 10: Capillary pressure curves for Case 2. (a) Capillary pressure curves showing the LSWF effect. The red curve is for HS, while the blue curve is for LS. Fitted Brooks-Corey $p_c(s_w)$ models for (b) HS (c) LS. The blue points represent the values calculated in the primary drainage algorithm. The orange points are obtained using the fitted model. The dashed orange line shows the fitted curve trend.

In Figure 11(a), at the LS stage start (solid line), the 72% water saturation points out that only the LS fluid can move through the network. In fact, the water relative permeability reaches 0.15, while the oil relative permeability is practically 0. Specifically, when $s_w = 72\%$, the oil saturation is 28%. This saturation value is known as critical oil saturation; that is, the saturation at which the oil begins to flow as the oil saturation increases. As water saturation
decreases, the water relative permeability also decreases, while the oil relative permeability increases. Another particular point on the relative permeability curve is $s_w = 26\%$, because at this saturation the water relative permeability becomes zero and the oil relative permeability is the maximum. This saturation is the critical water saturation and can be greater than or equal to the irreducible water saturation. At the LSWF end, the dashed line curves in Figure 11(a) show a slight shift to the right, compared to the initial curves (solid lines). Regarding the initial state, the system has a subtle change in wettability, the curve crossing point moves from $s_w = 0.66$ to 0.68. Moreover, prior the oil becomes mobile and comparing the initial state to the final, the water relative permeability endpoint moves from 0.15 to 0.40. Finally, the $k_r$ curves for HS and LS are fitted (Figure 11(b) and Figure 11(c)(c), respectively), and the fitting parameters are summarized in Table 5.
Figure 11: Relative permeability curves for Case 2. (a) The red lines show the oil relative permeability $k_{ro}$ and the blue lines the water relative permeability $k_{rw}$. Solid lines correspond to HS and dashed lines to LS, and original data are in red and blue. The curves were fitted at the end of the (b) HS and (c) LS stages. In curve fitting, a Brooks-Corey $k_{rw}$ model (green line) and a modified Brooks-Corey $k_{ro}$ model (pink line) were used.

6. Conclusions

We have introduced a pore network based methodology to study the salinity effect on capillary pressure and relative permeability curves. Similarly, we formulated, implemented, and validated an advective-diffusive salinity transport model for pore networks. This model computes the salinity concentration values both in pores and throats. Since throats control the capillary pressure, knowing
the throat salinity concentration allows us to evaluate the salinity impact on the
capillary pressure curves. The way we associate the salinity and the capillary
pressure is through the contact angle, which is a linear function of the salinity
concentration. The wettability of the network is modified by the changes in the
contact angle, which in turn depends on the salinity. By reducing salinity the
system becomes more water wet. By applying the methodology to two cases,
one for a sandstone sample and the other for a carbonate sample, it has been
demonstrated that our methodology can be used as a tool to rapidly estimate
effective flow properties, which is an advantage over experimental procedures.

Nomenclature

Subscripts

0 .... Initial
e .... Effective
i, j .. Pore index
ij ... Throat index
iny .. Injection
p .... Pore
t .... Throat

Greek symbols

κ .... Hydraulic conductance
μ .... Viscosity
θ .... Contact angle

Symbols

A .... Cross-sectional area
$D$ ... Diffusion coefficient

$I$ ... Pore and throat indexes

$K$ ... Absolute permeability

$L$ ... Network length

$l$ ... Throat length

$p$ ... Pressure

$Q$ ... Total flow

$q$ ... Flow rate

$r$ ... Radii

$t$ ... Time

$V$ ... Volume

**Superscripts**

$HS$ ... High salinity

$LS$ ... Low salinity

**Appendix A. Validation**

In this appendix the validation of the numerical code for the salinity convective-dispersive transport using a 1D network system is described. The analytical solution employed to validate Equation 5 and Equation 6 is that presented by van Genuchten & Alves (1982) for a pulse type injection. The equation is:

$$ R \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - V \frac{\partial c}{\partial x} $$

(A.1)

Equation A.1 is subjected to the initial and boundary conditions:

$$ c(x, 0) = c_0 $$

(A.2)
where, $t_{inj}$ and $c_{inj}$ are the injection time and the salinity concentration during an injection period, respectively. The concentration at initial time is denoted as $c_0$. The semi-analytical solution of Equation A.1 is:

$$c(x, t) = \begin{cases} 
  c_0 + (c_{inj} - c_0)A(x, t) & 0 < t < t_{inj} \\
  c_0 + (c_{inj} - c_0)A(x, t) - c_{inj}A(x, t) & t > t_{inj}
\end{cases}$$

(A.3)

where

$$A(x, t) = 1 - \sum_{m=1}^{\infty} \frac{2\beta_m \sin \left( \frac{\beta_m x}{L} \right) \exp \left( \frac{vx}{2D} - \frac{v^2 t}{4DR} - \frac{\beta_m^2 Dt}{L^2 R} \right)}{\left( \frac{\beta_m^2}{2D} + \left( \frac{vL}{2D} \right)^2 + \frac{vL}{2D} \right)}$$

(A.4)

and where the eigenvalues $\beta_m$ are the positive roots of the equation:

$$\beta_m \cot(\beta_m) + \frac{vL}{2D} = 0$$

(A.5)

In our validation, a 10-pore one-dimensional network and ideal conditions are established. The considered parameters are summarized in Table A.8. Three 1000-iteration simulations were run for different time steps $\Delta t$, here 1, 0.1, and 0.01 [s] were employed. In this way, 1000, 100, and 10 [s] were considered as simulation times. To compare obtained results, concentration values at the outlet boundary were plotted for each time step (Figure A.12).
Table A.8: Parameters used in validation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial concentration $c_0$</td>
<td>0</td>
<td>ppm</td>
</tr>
<tr>
<td>Injection concentration $c_{inj}$</td>
<td>200</td>
<td>ppm</td>
</tr>
<tr>
<td>Domain length $L$</td>
<td>1E-03</td>
<td>m</td>
</tr>
<tr>
<td>Injection time $s$</td>
<td>500, 50, and 5</td>
<td>s</td>
</tr>
<tr>
<td>Final time $t_f$</td>
<td>1000, 100, and 10</td>
<td>s</td>
</tr>
<tr>
<td>Time step $\Delta t$</td>
<td>1, 0.1, and 0.01</td>
<td>s</td>
</tr>
<tr>
<td>Injection velocity $v$</td>
<td>1.38E-04</td>
<td>m/s</td>
</tr>
<tr>
<td>Diffusion coefficient $D$</td>
<td>2.06E-08</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$R$</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Figure A.12: Concentration at the outlet boundary for different time steps. (a) $\Delta t = 1$ [s], (b) $\Delta t = 0.1$ [s], and (c) $\Delta t = 0.01$ [s]. The plots show a comparison between the numerical simulation (red line) and the semi-analytical solution (blue line).

The comparison between the numerical simulation and the semi-analytical solution is shown in Figure 12(a), Figure 12(b) and Figure 12(c), where the solution using $\Delta t = 1$, 0.1, and 0.01 seconds, is respectively shown. By decreasing time step, Equation 5 and Equation 6 approach to the semi-analytical model. However, when $\Delta t = 0.01$ [s], the curves become separated at $t > 5$ [s]. But, since the mixtures are perfect and the reactions are neglected within pores and throats, this work does not require considering time steps below the thousandths of a second. Finally, according to computed errors summarized in Table A.9, it is agreed that the model (Equation 5 and Equation 6) is validated, providing
approximate values to the semi-analytical solution are obtained.

Table A.9: Computed error (%) from the numerical simulation and the semi-analytical solution.

<table>
<thead>
<tr>
<th>Δt [s]</th>
<th>Absolute error</th>
<th>Relative error</th>
<th>Mean absolute error</th>
<th>Mean squared error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>812</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0.1</td>
<td>5</td>
<td>777</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>0.01</td>
<td>7</td>
<td>228</td>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

Appendix B. Modeling methodology using pore network models

In the literature, it has not been found a workflow that describes step by step the application of pore network models; however, common practices have been recognized. By following this review, fundamental procedures and techniques can be integrated and a methodology becomes apparent. It is noteworthy that each methodology stage can be divided into more fields, thus allowing a better study range (Martínez-Mendoza, 2016). In Figure B.13, the solid red line points out the proposed methodology. However, depending on the ongoing stage, deviations from the proposed flow can be followed (dotted black lines). For example, a pore network can be generated directly from data acquisition (extracted networks) without performing statistical analysis. Likewise, it is possible to create a network if the workflow starts from statistical analysis. Finally, after validation and uncertainty assessment, the workflow can turn back either data acquisition or statistical analysis in order to perform new realizations. The methodology stages are described below.

Data acquisition

In this stage, various properties that describe the porous medium, the fluids and the rock-fluid interactions are obtained. In pore network modeling, it becomes crucial to know the pore and the throat shapes and sizes, as well as the way in which these elements are connected. Therefore, the following pore and throat properties must be defined: spatial location, coordination number,
Figure B.13: Modeling methodology using pore networks. The methodology comprises five fundamental stages: data acquisition, statistical analysis of pore structure properties, pore network model, validation and uncertainty assessment, and applications.

area, inscribed diameter, perimeter, total length, volume, cross section, and shape factor. To characterize the porous medium there are high resolution imaging techniques such as scanning electron microscopy (1 – 20 [nm]), focused ion beam (< 1 [nm]), confocal laser scanning microscopy (~ 1 [µm]), X-ray computed micro-tomography (1 – 200 [µm]), and nuclear magnetic resonance (2 [nm] – 1 [µm]) (Dong, 2007; Xiong et al., 2016). Applying these image techniques, the pore and the throat sizes, the network node coordinates and their connections with other nodes are inferred. Finally, to characterize the fluids and the rock-fluid system, laboratory tests to measure density, viscosity, interfacial tension, and contact angle, among other properties, are performed.

**Statistical analysis of pore structure properties**

This stage involves performing statistical analysis to main medium structure properties (e.g., throat and pore sizes, and coordination number) to fit probability distribution functions. The probability distribution functions generate the aforementioned property values, and consequently, a porous media analogous
to the original one can be achieved. In addition, performing network spatial statistical analysis to infer connectivity and percolation properties is highly recommended. With all this information it is possible different spatial realizations of the original sample.

**Pore network model**

From the previous stage an optimal framework for setting the fundamental pore network elements can be established. The complexity and detail with which the network and geometry are represented establish a starting point to formulate the pore-scale physics and consequently, the method scope. In principle, the pore network models can be generated from three-dimensional reconstructions of rock samples and from their respective extracted network and geometry. Although this technique for pore network generation is established, it results in reality that the required information and tools are not fully available. Further, working with a unique extracted network could yield a narrow study window, and the model cannot be representative. Hence, the statistical analysis represents an attractive option to generate different realizations for the network and the geometry. Finally, the simulations using these models can help to build a reliability window, which provides a value range that a desired property can take for a particular medium (e.g., absolute permeability, capillary pressure, relative permeability).

**Validation and uncertainty assessment**

The validations of properties such as porosity, absolute permeability, capillary pressure, and relative permeability, can be achieved by comparing them against the corresponding laboratory test data. This comparison allows us to verify that the implemented pore network model is representative of the phenomena. In addition, an uncertainty evaluation can be obtained by multiple simulations exploring the pore network parameters (e.g., the throat and pore size distributions).
Finally, the obtained porosity, absolute permeability, capillary pressure, and relative permeability values can be used in subsequent analyzes, such as curve fitting, multiphase flow models, simulations at larger scales, and multiscale simulations.

References


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Highlights

- The effect of low salinity water flooding (LSWF) on capillary pressure and relative permeability is studied.
- An advective-diffusive salinity transport model for pore networks is developed.
- A pore network based methodology to describe LSWF processes is established.
- This methodology can be used as a tool to rapidly estimate effective flow properties.